Nonlinear Diffusions and Entropy Dissipation: From Geometry to Biology
May 9 - 14, 2010.

MEALS

*Breakfast (Buffet): 7:00-9:30 am, Sally Borden Building, Monday-Friday
*Lunch (Buffet): 11:30 am-1:30 pm, Sally Borden Building, Monday-Friday
*Dinner (Buffet): 5:30-7:30 pm, Sally Borden Building, Sunday-Thursday
***Please remember to scan your meal card at the host/hostess station in the dining room for each meal.**

MEETING ROOMS

All lectures will be held in Max Bell 159 (Max Bell Building accessible by walkway on 2nd floor of Corbett Hall). LCD projector, overhead projectors and blackboards are available for presentations. Further meeting space designated for BIRS is the lower level of Max Bell, Rooms 155-159. All other space has been contracted to other Banff Centre guests.

SCHEDULE

Sunday

16:00	Check-in begins (Front Desk - Professional Development Centre - open 24 hours)
	Lecture rooms available after 16:00 (if desired)
17:30-19:30	Buffet Dinner, Sally Borden Building
20:00	Informal gathering in 2nd floor lounge, Corbett Hall
	Beverages and a small assortment of snacks are available on a cash honor system.

Monday

7:00-8:40	Breakfast
8:40-8:50	Introduction and Welcome by BIRS Station Manager, Max Bell 159
8:50-9:40	Robert McCann
	Higher order asymptotics of fast diffusion in Euclidean space
9:40-10:10	Martial Agueh
	Finsler structure in the p-Wasserstein space and applications to PDEs
10:10-10:40	Coffee Break, 2nd floor lounge, Corbett Hall
10:40-11:30	Giuseppe Savaré
	Mobility functions, transport distances, and nonlinear diffusion
11:30-12:00	Bruno Nazaret
	Distances on probability measures induced by concave mobilities : geodesics via PDE
12:00-13:00	Lunch (until 13:55 for those who do not going on the Guided Tour)
13:00-13:55	Guided Tour of The Banff Centre; meet in the 2nd floor lounge, Corbett Hall
13:55-14:00	Group Photo; meet on the front steps of Corbett Hall
14:00-14:50	Matteo Bonforte
	Asymptotics of the Fast Diffusion Equation via Entropy Methods
14:50-15:20	Christian Schmeiser
	Hypocoercivity of a kinetic model for fast diffusion
15:20-15:50	Coffee Break, 2nd floor lounge, Corbett Hall
15:50-16:40	Alessio Figalli
	A new transportation distance between non-negative measures
17:30-19:30	Dinner

Tuesday

7:00-8:50	Breakfast
8:50-9:40	Panagiota Daskalopoulos *Ancient solutions to the Ricci flow on Surfaces*
9:40-10:10	Paul Woon Yin Lee *Generalized Ricci Curvature Bounds for Three Dimensional Contact Subriemannian Manifolds*
10:10-10:40	Coffee Break, 2nd floor lounge, Corbett Hall
10:40-11:30	Nataša Šešum *Curvature flows*
11:30-12:00	Michael Westdickenberg *Optimal Transport for the System of Isentropic Euler Equations*
12:00-13:40	Lunch
13:40-14:30	Karl-Theodor Sturm *Monotone Approxiamtion for the Wasserstein Diffusion*
14:30-15:00	Thomas Laurent *Mechanism of formation of Dirac masses in the solutions of the aggregation equation*
15:00-15:30	Coffee Break, 2nd floor lounge, Corbett Hall
15:30-16:00	Laurent Desvilletes *Entropy methods and cross diffusions*
16:00-16:30	Marco Di Francesco *On the Hughes' model for pedestrian flow: The one-dimensional case*
16:30-17:00	Jesus Rosado *A refined result of flocking for the Cucker-Smale model*
17:30-19:30	Dinner

Wednesday

7:00-8:50	Breakfast
8:50-9:40	David Kinderlehrer *Coarsening in cellular systems*
9:40-10:10	Robert Stańczy *Entropy methods for self-gravitating particles systems*
10:10-10:40	Maria del Mar Gonzalez *A free-boundary model in price formation*
10:40-11:00	Coffee Break, 2nd floor lounge, Corbett Hall

Thursday

7:00-8:50	Breakfast
8:50-9:40	Adrien Blanchet *Functional inequalities, thick tails and asymptotics for the critical mass Patlak-Keller-Segel model*
9:40-10:10	Lucilla Corrias *The role of the free energy and its dissipation in the mathematical analysis of the parabolic-parabolic Keller-Segel system*
10:10-10:40	Coffee Break, 2nd floor lounge, Corbett Hall
10:40-11:10	Vincent Calvez *Gradient flow interpretation for the one-dimensional Keller-Segel equation*
11:10-11:40	Piotr Biler *Keller-Segel systems at* $M = 8\,\pi$
11:40-12:10	Philippe Laurencot *Large time behaviour for quasilinear a degenerate diffusion equation with gradient source term*
12:10-13:40	Lunch
13:40-14:30	Dominique Bakry *Weighted Nash Inequalities*
14:30-15:00	Daniel Matthes *Gradient flow of the Dirichlet functional*
15:00-15:30	Coffee Break, 2nd floor lounge, Corbett Hall
15:30-16:20	Klemens Fellner *On non-local interaction equations: Stability of Stationary States and Limiting Behaviour*
16:20-16:50	Gael Raoul *Selection dynamics for the evolution of traits in a population*
17:30-19:30	Dinner

Friday

7:00-8:50	Breakfast
8:50-9:40	Marina Chugunova *Entropy-energy analysis of coating flows*
9:40-10:10	Suleyman Ulusoyi *Long Time Behavior of Weak Solutions in Hele-Shaw Flow Problem*
10:10-10:40	Coffee Break, 2nd floor lounge, Corbett Hall
10:40-11:30	José Alfredo Canizo *Entropy-entropy dissipation inequalities for fragmentation-drift equations*
12:00-14:00	Lunch

Checkout is by 12:00. BIRS facilities (2nd Floor Lounge, Max Bell Meeting Rooms, Reading Room) are available until 3 pm on Friday.

AGUEH, MARTIAL

Finsler structure in the p-Wasserstein space and applications to PDEs

It is known from the work of F. Otto [*Comm. Partial Diff. Eq. 2001*], that the space of probability measures equipped with the quadratic Wasserstein distance, i.e., the 2-Wasserstein space, can be viewed as a "Riemannian manifold". Here we show that when the quadratic cost function is in general replaced by a homogeneous cost of degree $p > 1$, then the corresponding space of probability measures, i.e., the p-Wasserstein space, can be endowed with a Finsler metric whose induced distance function is the p-Wasserstein distance. Using this Finsler structure of the p-Wasserstein space, we give definitions of the differential and gradient of functionals defined on this space, and then of gradient flows in this space. In particular we show in this framework that the parabolic q-Laplacian equation is a gradient flow in the p-Wasserstein space, where $p = q/(q-1)$. When $p = 2$, we recover the Riemannian structure introduced by Otto, which confirms that the 2-Wasserstein space is a Riemann-Finsler manifold. Our approach is confined to a smooth situation where probability measures are absolutely continuous with respect to the Lebesgue measure on R^n, and they have smooth and strictly positive densities.

BAKRY, DOMINIQUE

Weighted Nash Inequalities

In this talk, we shall present some families of weighted Nash type inequalities, which lead to non uniform upper bound to heat kernels. This may produce estimates on the Hilbert-Schmidt norm for Markov semigroups. We shall show on some 1-dimensional model examples that those inequalities are quite easy to obtain, even in the case where no ultracontractive or even hypercontractive bounds may be expected.

BILER, PIOTR

Keller-Segel systems at $M = 8\,\pi$

We consider models for chemotaxis

$$n_t = \Delta n - \nabla \cdot (n \nabla c)\,,$$

$$\tau\, c_t = \Delta c + n\,,$$

for $x \in \mathbb{R}^2$, $t > 0$, and for each $\tau \geq 0$.

As it is well known for the parabolic-elliptic system ($\tau = 0$), there is infinite number of locally stable steady states of critical mass $M = 8\pi$. On the other hand, initially not dispersed data lead to solutions that concentrate (or, in other words, blow up) in the infinite time: $n(\cdot\,,t) \rightharpoonup 8\,\pi\,\delta$ as $t \to \infty$.

The picture is much more complicated for the doubly parabolic Keller-Segel model ($\tau > 0$) since for each $\tau > \tau_*$ (with some $\tau_* > 0$), besides those steady states, there is a self-similar solution (n, c) of the form

$$\left(\frac{1}{t} u\left(\frac{x}{\sqrt{t}}\right), v\left(\frac{x}{\sqrt{t}}\right)\right)$$

also of mass $M = 8\,\pi$, which decays exponentially for $|x| \to \infty$. For this self-similar solution whole mass diffuses to infinity as $t \to \infty$.

BLANCHET, ADRIEN

Functional inequalities, thick tails and asymptotics for the critical mass Patlak-Keller-Segel model

We investigate the long time behaviour of the critical mass Patlak-Keller-Segel equation. This equation has a one parameter family of steady-state solutions $\rho_{\infty,\lambda}$, $\lambda > 0$, with thick tails whose second moment is not bounded. We show that these steady state solutions are stable, and find basins of attraction for them using an entropy functional $\mathcal{H}$ coming from the critical fast diffusion equation in $\mathbb{R}^2$. We construct solutions of Patlak-Keller-Segel equation satisfying an entropy-entropy dissipation inequality for $\mathcal{H}$. While the entropy dissipation for $\mathcal{H}$ is strictly positive, it turns out to be a difference of two terms, neither of which need to be small when the dissipation is small. We introduce a strategy of *controlled concentration* to deal with this issue, and then use the regularity obtained from the entropy-entropy dissipation inequality to prove the existence of basins of attraction for each stationary state composed by certain initial data converging towards $\rho_{\infty,\lambda}$. In the present talk, we do not provide any estimate of the rate of convergence, but we discuss how this would result from a stability result for a certain sharp Gagliardo-Nirenberg-Sobolev inequality.

BONFORTE MATTEO

Asymptotics of the Fast Diffusion Equation via Entropy methods

We consider non-negative solutions of the fast diffusion equation $u_t = \Delta u^m$ with $m \in (0,1)$, in the Euclidean space $\mathbb{R}^d$, $d \geq 3$, and study the asymptotic behavior of a natural class of solutions, in the limit corresponding to $t \to \infty$ for $m \geq m_c = (d-2)/d$, or as t approaches the extinction time when $m < m_c$. For a class of initial data we prove that the solution converges with a polynomial rate to a self-similar solution, for t large enough if $m \geq m_c$, or close enough to the extinction time if $m < m_c$. Such results are new in the range $m \leq m_c$ where previous approaches fail. In the range $m_c < m < 1$ we improve on known results.

The precise value for the exponent $m = (d-4)/(d-2)$, $d \geq 3$, requires quite different functional analytic methods, due in particular to the absence of a spectral gap for the linearized generator.

CALVEZ, VINCENT

Gradient flow interpretation for the one-dimensional Keller-Segel equation.

Joint work with J.A. Carrillo (ICREA, UAB, Barcelona, Spain).

We wish to present a brief overview of recent developments in the analysis of the generalized Keller-Segel equation in dimension one:

$$\partial_t \rho(t,x) = \partial_{xx}\rho^m(t,x) + \chi\partial_x\left(\rho(t,x)\partial_x\left(\frac{|x|^k}{k} * \rho(t,x)\right)\right), \quad t > 0, \quad x \in \mathbb{R},$$

for the following set of exponents: $m \geq 1$ and $k \in (-1,1)$. Here $\rho(t,x)$ denotes a density of cells which diffuse and attract each other through a mean field potential. This model has raised a lot of interest in the field of mathematical biology since it generally exhibits a dichotomy between global existence (dispersion of the cells) and finite time blow-up (aggregation of the cells).

This equation possesses the structure of a gradient flow for the Wasserstein metric on the space of probability densities. Interestingly enough the energy functional $\mathcal{F}$ is the sum of two opposite contributions, being respectively displacement-convex and concave, and homogeneous. In particular we obtain some blow-up criterion as a direct consequence of the homogeneity property in the case $m + k < 1$ (*attraction-dominating competition*):

$$\mathcal{F}\left[\rho_0\right] < C\frac{(m-1+k)}{k}\left(\int_{\mathbb{R}} |x|^2\rho_0(x)\,dx\right)^{(1-m)/2}.$$

On the other hand we investigate the long-time asymptotics in the case $m + k = 1$ (*fair competition*). We are able to prove the following inequalities along the gradient flow in appropriately rescaled variables:

$$\frac{1}{2}\frac{d}{dt}W\left(\rho(t),\mu\right)^2 + W(\rho(t),\mu)^2 \le 0\,, \quad (\mathbf{m=1})\,,$$

$$\frac{1}{2}\frac{d}{dt}W\left(\rho(t),\mu\right)^2 + \frac{m+1}{2}W(\rho(t),\mu)^2 \le (m-1)\left(\mathcal{F}[\rho(t)] - \mathcal{F}[\mu]\right)\,, \quad (\mathbf{m>1})\,,$$

where μ denotes the minimizer of the energy functional (supplemented with a quadratic confinement potential due to rescaling). We deduce from the former a uniform rate of convergence towards the minimizer in the case $(m = 1, k = 0)$. These inequalities are strongly related to a convex-like property of the energy functional.

CANIZO, JOSÉ ALFREDO

Entropy-entropy dissipation inequalities for fragmentation-drift equations

The growth-fragmentation equation models a group of cells which grow (or age) at a certain rate, and divide into two or more pieces. After a suitable rescaling, the shape of the distribution function of the population converges to a universal profile. By means of entropy-entropy dissipation inequalities for some unbounded fragmentation coefficients, we show that the speed of this convergence is exponential. This kind of inequalities may be used for other linear equations for which the General Relative Entropy principle is valid.

CHUGUNOVA, MARINA

Entropy-energy analysis of coating flows

The equation

$$u_t + [u^n(u_{xxx} + \gamma^2 u_x - \sin(x))]_x = 0$$

with periodic boundary conditions is a model of the evolution of a thin liquid film on the outer surface of a horizontal cylinder in the presence of gravity field. We use entropy-energy methods to study different properties of generalized weak solutions of this equation. For example: finite speed of the compact support propagation for $n \in (1, 3)$ is proved by application of local α-entropy estimates.

Joint work with A. Burchard, M. Pugh, B. Stephens, and R. Taranets

CORRIAS, LUCILLA

The role of the free energy and its dissipation in the mathematical analysis of the parabolic-parabolic Keller-Segel system.

It is well known that the classical Keller-Segel system modeling chemotaxis is endowed with a free energy functional decreasing along the trajectories of the solutions. In this talk, we shall show how to handle that energy with the help of several sharp functional inequalities, in order to obtain both global existence results and density concentration phenomena for the fully parabolic Keller-Segel system on the whole space. The case of the high space dimension is also considered. (Joint works with V. Calvez and A. Ebde).

DASKALOPOULOS, PANAGIOTA

Ancient solutions to the Ricci flow on Surfaces

We provide a classification for ancient solutions to the Ricci flow on S^2. We show that they are either the contracting spheres or the King-Rosenau solutions. We also prove a similar result for ancient convex embedded solutions to the curve shortening flow. This is joint work with Richard Hamilton and Natasa Sesum.

DESVILLETTES, LAURENT

Entropy methods and cross diffusions

Some models in population dynamics include so-called cross diffusions terms. Those terms model the effect of the presence of one type of individuals on the diffusion rate of other individuals. We shall present in this talk results about the modeling and the mathematical analysis of systems including cross diffusion: those results share a common feature: the use of Lyapounov-like functionals (entropies).

DI FRANCESCO, MARCO

Nonlinear Cross-Diffusion with Size Exclusion

In this talk we investigate the mathematical theory of Hughes' model for the flow of pedestrians (cf. Hughes 2002), consisting of a nonlinear conservation law for the density of pedestrians coupled with an eikonal equation for a potential modelling the common sense of the task. We first consider an approximation of the original model in which the eikonal equation is replaced by an elliptic approximation. For such an approximated system we prove existence and uniqueness of entropy solutions (in one space dimension) in the sense of Kruzkov, in which the boundary conditions are posed following the approach of Bardos et al.. We use BV estimates on the density and stability estimates on the potential in order to prove uniqueness. Furthermore, we analyse the evolution of characteristics for the original Hughes' model in one space dimension and study the behaviour of simple solutions, in order to reproduce interesting phenomena related to the formation of shocks and rarefaction waves. The characteristic calculus is supported by numerical simulations.

FELLNER, KLEMENS

On non-local interaction equations: Stbility of Stationary States and Limiting Behaviour

We study non-local evolution equations for a density of individuals, which interact through a given symmetric potential. Such models appear in many applications such as swarming and flocking, opinion formation, inelastic materials, ... In particular, we are interested in interaction potentials, which behave locally repulsive, but aggregating over large scales. A particular example for such potentials was recently given in models of the alignment of the directions of filaments in the cytoskeleton.

We present results on the structure and stability of steady states. We shall show that stable stationary states of regular interaction potentials generically consist of sums of Dirac masses. However the amount of Diracs depends delicately on the interplay between local repulsion and aggregation. In particular we shall see that singular repulsive interaction potentials introduce diffusive effects in the sense that stationary state are rendered continuously.

FIGALLI, ALESSIO

A new transportation distance between non-negative measures

Given a bounded domain $\mathcal{O}$, it is by now well-known that that the gradient flow of the entropy functional $\int_{\mathcal{O}}[\rho\log(\rho)-\rho]\,dx$ with respect to the Wasserstein distance produces a solution to the heat equation with Neumann boundary conditions. Recently, in collaboration with Nicola Gigli we introduced a new transportation distance between non-negative measures inside a domain $\mathcal{O}$. This distance enjoys many nice properties, for instance it makes the space of non-negative measures inside $\mathcal{O}$ a geodesic space, without any convexity assumption on $\mathcal{O}$. Moreover, the gradient flow of the entropy functional $\int_{\mathcal{O}}[\rho\log(\rho)-\rho]\,dx$ w.r.t. this distance coincides with the heat equation subject to the Dirichlet boundary condition equal to 1. The aim of this talk will be to briefly review the classical theory, and then to introduce this new distance and its main properties.

GONZALEZ, MARIA DEL MAR

A free-boundary model in price formation

We study a model, due to J.M. Lasry and P.L. Lions, describing the evolution of a scalar price which is realized as a free boundary in a 1D diffusion equation with dynamically evolving, non-standard sources. We establish global existence and uniqueness. This is joint work with L. Chayes, M. Gualdani and I. Kim.

KINDERLEHRER, DAVID

Coarsening in cellular systems

Mesoscale experiment and simulation permit harvesting information about both geometric features and texture in material microstructures. The grain boundary character distribution (GBCD) is an empirical distribution of the relative length (in 2D) or area (in 3D) of interface with a given lattice misorientation and grain boundary normal. During the growth process, an initially random texture distribution reaches a steady state that is strongly correlated to the interfacial energy density. In simulation, we found that if the given energy depends only on lattice misorientation, then the steady state GBCD and the energy are related by a Boltzmann distribution. This is among the simplest non-random distributions, corresponding to independent trials with respect to the energy. Why does such a simple distribution arise from such a complex system?

We outline an entropy based theory which suggests that the evolution of the GBCD satisfies a Fokker-Planck Equation. Coarsening in polycrystalline systems is a complicated process involving details of material structures, chemistry, arrangement of grains in the configuration, and environment. In this context, we consider just two global features: cell growth according to a local evolution law of curvature driven growth and space filling constraints. Space filling requirements are managed by critical events, rearrangements of the network involving deletion of small contracting cells and facets. The interaction between the evolution law and the constraints is governed primarily by the Herring Condition, the boundary condition associated with the equation of curvature driven growth. It determines a dissipation relation. To assist in the derivation, a simpler system is introduced which is driven by the boundary conditions and reflects the dissipation relation of the grain growth system. It resembles an ensemble of inertia-free spring-mass-dashpots. For this simpler coarsening network, we learn how entropic or diffusive behavior at the large scale emerges from a dissipation relation at the scale of local evolution.

Joint work with K. Barmak, E. Eggeling, M. Emelianenko, Y. Epshteyn, R. Sharp, and S. Ta'asan.

LAURENCOT, PHILIPPE

Large time behaviour for quasilinear a degenerate diffusion equation with gradient source term

The qualitative behaviour of nonnegative solutions to the Cauchy-Dirichlet problem for the equation $\partial_t u - \Delta_p u = |\nabla u|^q$ in a bounded domain depends on the relative strength of the diffusion and the source terms. According to the values of p and q, multiple steady states may exist or convergence to a "friendly giant" may take place or finite time blowup may occur (joint works with G. Barles, C. Stinner, and M. Winkler).

LAURENT THOMAS

Mechanism of formation of Dirac masses in the solutions of the aggregation equation

The aggregation equation is a continuum model for interacting particle systems with attractive/repulsive pairwise interaction potential. It arises in a number of models for biological aggregation, but also in materials science and granular media. As a mathematics problem, the aggregation equation is a gradient flow of the interaction energy with respect to the Wasserstein distance. The main phenomenon of interest is that, even with smooth initial data, the solutions can concentrate mass in finite time (i.e. a delta Dirac appears in the solution in finite time). We study the mechanism of formation of these delta Dirac masses.

LEE, PAUL WOON YIN

Generalized Ricci Curvature Bounds for Three Dimensional Contact Subriemannian Manifolds

Measure contraction property (MCP) is one of the possible generalizations of Ricci curvature bound to more general metric measure spaces. However, the definition of MCP is not computable in general. In this talk, I'll discuss computable sufficient conditions for a three dimensional contact subriemannian manifold to satisfy such property. This is a joint work with Andrei Agrachev.

MATTHES, DANIEL

Gradient flow of the Dirichlet functional

On a formal level, the Cahn-Hillard equation $\partial_t u = -\mathrm{div}\big(\mathbf{m}(u)\nabla\Delta u\big) + \Delta P(u)$ is readily seen to constitute a gradient flow of a perturbed Diriclet energy functional. The properties of the associated metric – which is a Wasserstein-like distance, but with the *non-linear mobility* function $\mathbf{m}$ – have been investigated recently by Dolbeault, Nazareth and Savar. In this talk, we show how their results can be used to make the formal variational structure of the Cahn-Hilliard equation rigorous. Our basic assumption is that $\mathbf{m}$ is concave and satisfies some growth condition.

In application of the rigorous gradient flow structure, we derive sharp estimates on the intermediate asymptotics of weak solutions to the Hele-Shaw flow. The latter corresponds to the Cahn-Hilliard equation with *linear* mobility $\mathbf{m}$, when the associated metric is just the classical L^2-Wasserstein distance. The key idea is to use a very particular connection between the Diriclet energy, an entropy and the Wasserstein distance.

This is joint work with Stefano Lisini and Giuseppe Savar (Pavia).

MCCANN, ROBERT

Higher order asymptotics of fast diffusion in Euclidean space

With Denzler and Koch, we quantify the speed of convergence and higher-order asymptotics of fast diffusion dynamics on $\mathbf{R}^n$ to the Barenblatt (self similar) solution. Degeneracies in the parabolicity of this equation are cured by re-expressing the dynamics on a manifold with a cylindrical end, called the cigar. The nonlinear evolution semigroup becomes differentiable in Hölder spaces on the cigar. The linearization of the dynamics is given by the Laplace-Beltrami operator plus a transport term (which can be suppressed by introducing appropriate weights into the function space norm), plus a finite-depth potential well with a universal profile. In the limiting case of the (linear) heat equation, the depth diverges, the number of eigenstates increases without bound, and the continuous spectrum recedes to infinity. We provide a detailed study of the linear and nonlinear problems in Hölder spaces on the cigar, including a sharp boundedness estimate for the semigroup, and use this as a tool to obtain sharp convergence results toward the Barenblatt solution (as Bonforte, Dolbeault, Grillo and Vazquez also did independently in a weaker metric), and higher order asymptotics. In finer convergence results (after modding out symmetries of the problem), a subtle interplay between convergence rates and tail behavior is revealed. The difficulties involved in choosing the right functional spaces in which to carry out the analysis can be interpreted as genuine features of the equation rather than mere annoying technicalities.

NAZARET, BRUNO

Distances on probability measures induced by concave mobilities : geodesics via PDE

In the framework of distances on probability measures introduced by Dolbeault, Nazaret and Savaré, we present some cases where the geodesics are obtained as solutions to a system of PDE on the density and the velocity of the intermediate configurations (joint work with F. Santambrogio).

RAOUL GAEL

Selection dynamics for the evolution of traits in a population

We consider a model describing the evolution of a large population of individuals with different biological "traits". The reproduction rate of each individual depends on its trait and on the competition for resources within the population. For a certain class of coefficients, an entropy exists, and can be used to study the large time behaviour of the population, and in particular the formation of Dirac masses, that corresponds to species formation.

ROSADO, JESUS

A refined result of flocking for the Cucker-Smale model

I will present and analyse the asymptotic behaviour of solutions of the continuous kinetic version of flocking by Cucker and Smale [2], which describes the collective behaviour of an ensemble of organisms, animals or devices. This kinetic version introduced by Ha and Tadmor is obtained from a particle model. The large-time behaviour of the distribution in phase space is subsequently studied by means of particle approximations and a stability property in distances between measures. A continuous analogue of the theorems of Cucker and Smale will be shown to hold for the solutions on the kinetic model. More precisely, the solutions concentrate exponentially fast their velocity to their mean while in space they will converge towards a translational flocking solution. The presentation is based on [1].

SAVARÉ, GIUSEPPE

Mobility functions, transport distances, and nonlinear diffusion

We present a new class of distances between nonnegative measures in the Euclidean spaces. They are modeled on the dynamical characterization of the Wasserstein distances proposed by Benamou-Brenier and provide a wide family interpolating between the Wasserstein and the homogeneous Sobolev distances.

From the point of view of optimal transport theory, these distances minimize a dynamical cost to move a given initial distribution of mass to a final configuration. An important difference with the classical setting in mass transport theory is that the cost not only depends on the velocity of the moving particles but also on the densities of the intermediate configurations with respect to a given reference measure.

New existence, stability, and contraction results for solutions of nonlinear diffusion equations in a convex bounded domain will be discussed. (In collaboration with J. Dolbeault, B. Nazaret, J. Carrillo, S. Lisini, D. Slepcev)

SCHMEISER, CHRISTIAN

Hypocoercivity of a kinetic model for fast diffusion

Convergence to equilibrium will be discussed for BGK kinetic models, which have fast diffusion equations as their macroscopic limits.

ŠEŠUM, NATAŠA

Curvature flows

We will discuss various curvature flows such as the harmonic mean curvature flow (HMCF), the mean curvature flow (MCF) and the Ricci flow (RF). We will discuss the long time behaviour of the HMCF in the case of star shaped surfaces and also weakly convex hypersurfaces in higher dimensions. We will also discuss the curvature conditions that guarantee the smooth existence of the MCF and the RF.

STAŃCZY, ROBERT

Entropy methods for self-gravitating particles systems

Systems describing the interaction of gravitationally attracting particles that obey some statistics are studied. The role of the entropy for these system is thoroughly analyzed both in the isothermal case and in the case with the energy fixed. Systems of a similar form and structure appear also in modelling the chemotaxis phenomena in biology.

STURM, KARL-THEODOR

Monotone Approxiamtion for the Wasserstein Diffusion

We present a system of interacting Brownian particles on the real line for which the empirical distributions – in the scaling limit of large particle numbers – converge to the Wasserstein diffusion. The latter is a reversible Markov process with continuous paths on the space of probability measures whose square field operator – which governs the short time behavior – is the squared norm of the Wasserstein gradient. We indicate also some extensions to higher dimensional spaces.

ULUSOY, SULEYMAN

Long Time Behavior of Weak Solutions in Hele-Shaw Flow Problem

We investigate the long-time behavior of weak solutions to the thin- film type equation $u_t = -(uu_{xxx})_x$. We employ a semidiscrete variational scheme to generate weak solutions as a gradient flow with respect to so called Wasserstein distance and we show that these weak solutions converge to the unique self-similar source type solution exponentially fast. This paper complements our results in Carlen, E. A., Ulusoy, S. : Asymptotic equipartition and long time behavior of solutions of a thin-film equation, J. Differential Equations., 241, pp. 279-292, 2007. This is a joint work with Eric A. Carlen.

WESTDICKENBERG, MICHAEL

Optimal Transport for the System of Isentropic Euler Equations

The isentropic Euler equations form a system of conservation laws modeling compressible fluid flows with constant thermodynamical entropy. Due to the occurence of shock discontinuities, the total energy of the system is decreasing in time. We review the second order calculus on the Wasserstein space of probability measures and show how the isentropic Euler equations can be interpreted as a steepest descent equation in this framework. We introduce a variational time discretization based on a sequence of minimization problems, and show that this approximation converges to a suitably defined measure-valued solution of the conservation law. Finally, we present some preliminary results about the numerical implementation of our time discretization.